Radicals

Radicals are used to represent roots of numbers. The most common one represents a square root.

This represents the ***square root*** of *x*. This means that this expression is equal to whatever number can be multiplied by itself (or *square*, raised to the power of 2) will give you *x*.

For example: (we read this “the square root of 9 is equal to 3”) because

The 9 in this example is called the ***radicand***, and the shape around it is called the ***radical***. There is also a hidden number called the ***index***. If the index is not shown, *then it is understood to be a 2*.

So, the complete notation for this example would be . The small 2 in the upper part of the radical is the index.

If the index is larger than 2, we must write it. Let’s look at an example of an index larger than 2.

This represents the *cubed root* of 64. That is, this expression is equal to a number that, when raised to the 3rd power, gives you 64. We used the 3rd power because 3 is the index, just like we used the 2nd power in the previous example because the index was 2.

Notice that for an index of 2 and 3, we use the terms *square root* and *cube root*. For any index higher than 3, we would use ordinal number to describe them.

For example, would be read “the fourth root of 81.”  
And, would be read “the seventh root of 128.”

One other important fact about radicals has to do with the sign.

Let’s think of a number that, when squared, gives us 4. The number 2 automatically comes to mind. We know that . What often does *not* come to mind is -2. But, we should also realize that

as well.

So, to be precise about our answers, we would use the ± sign (sometimes call the plus-minus sign or positive negative sign).

So, for example, . This only applies to radicals with an even-numbered index.

Simplifying Radicals

Some radical expressions are easy to simplifying by simply recalling what you know from multiplying numbers in the past. For example, most people know that the square root of 4 is 2 (). Some other ones most people know or come to recognize with a little practice are:

Many other square roots of the numbers 1-10 are usually easy to recognize. However, sometimes we are not able to simplify radicals via our memories, so there is a process we follow to do this.

Let us start with an example we already know the answer to: . We know that the square root of 25 is 5 because 5 squared is 25. But, how can we find this out if we did not already know this?

We would start by finding the prime factorization of 25 and write it in condensed form using exponents.

Now, remember that there is an understood index of 2 in the radical.

Also, remember that this expression describes some number that, when squared, gives us the radicand. The radicand in this case is 52. Logically, we know that 5 raised to the power of two is 52, so our answer is 5.

Another way to look at it is to say that the exponent of the radicand cancels with the index.

Let’s look at a longer example.

Find the prime factorization in condensed form.

We can rewrite it in this way:

Now we can cancel the exponents with the index.

This is read “ten square root of two,” and is

equivalent to 10 times the square root of 2

Notice here that, when the index cancels the exponents in the radicand, the index does not go away if there is still anything left under it (the 2 with no exponent, in this case). Also notice that only the bases with the canceled exponents come out of the radical.

Another way to look at some of these steps includes the fact that factors in the radicand can be separated, and that factors can be multiplied in any order.

Now we can cancel the radicals whose indexes match the exponents of their radicands.

The Imaginary Number, *i*

(This is a very simple introduction to this concept that is here just so it can be used in the next section.)

Remember from above that when we have a square root, the answer can be positive or negative. This is because a negative number times a negative number produce a positive number. This begs the question, then, “What is the square root of a negative number?”

Think about it: What can you multiply by itself to get -4? A positive gives a positive 4, and also gives a positive number. There is no *real* number that can be squared to get a negative 4.

Because of this, mathematicians use what they call *the imaginary number i*.

With this *imaginary number*, we can say that

Likewise, we can say that: , , , and so on.

If we simplify a radical that ends up with a number on the outside and inside of the radical, it would look like this:

The *i* in this case is outside of the radical.

An interesting and important fact about this number *i* is that there is a cycle of it exponents that revert it back to itself.

So, you see that , so it goes through this cycle to turn back into itself.

Using and Simplifying Quadratic Formula

There are many methods of factoring quadratic expressions, each one designed to work with expressions in different forms and with different properties. The ***quadratic formula*** gives us a method that works with a braid range of expressions. The only requirement is that the expression is in the form below.

In this expression, the variables *a*, *b*, and *c* are constants (actual number as opposed to variables).

In this form, we can simply plug in the constants into the quadratic formula and simplify.

This is the Quadratic formula. Notice that it has all of the same variables as the expression above

(*x*, *a*, *b*, and *c*). When we plug the constant values into the right side of the formula and simplify, we are left with *x*.

Let’s try an example.

What are our variable values? , ,

Now plug them into the formula and simplify.

So, *x* is equal to either 2 or -8. You can plug these numbers into the original equation to test them. Sometimes, both answers are good answers, and sometimes only one of them is.

Let’s look at another example that uses *i*.

Identify the variable values. , ,

Plug them in.

Simplify.

Factor a two out of each term.

So, *x* is equal to and/or .